De Broglie's Wavelength

13.1.5-13.1.7

De Broglie's Wavelength

- 1923: De Broglie Hypothesis
  - There is a wave (wavelength = λ) that can be associated with any particle that has a momentum ***p***.
  - Duality of Matter: the concept that there are wave-like properties assigned to all matter that is normally treated only as a particle.

Example question:

- Find the de Broglie wavelength of a proton that has been accelerated from rest by a potential difference of 500. V.
  - Step 1: Remember that the work done to accelerate a charge through a potential difference is: $W = qV$
    - According to the work-energy theorem, the work done to a particle equals its change in energy, so:

$$\Delta E_k = qV$$

Example, continued…

- Step 2: the kinetic energy can be related to the momentum of a particle:

$$E_k = \frac{p^2}{2m}$$

  - Therefore: $\frac{p^2}{2m} = qV$

- Using de Broglie's Hypothesis, we see:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

Solution:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \text{ x } 10^{-34} J \cdot s}{\sqrt{2(1.67 \text{ x } 10^{-27} kg)(1.60 \text{ x } 10^{-19} C)(500\ V)}}$$

$$\lambda = 1.28 \text{ x } 10^{-12} m$$

Wave properties of an electron

- When does an electron exhibit wave-like properties?
  - Electrons display diffraction patterns when they pass through an opening that is comparable to (or smaller) than its de Broglie wavelength
- So the fundamental question really is: "What is an electron's de Broglie wavelength?"
  - Approximate velocity of a typical electron = 10^5 m·s^{-1}
  - Therefore, its momentum = 9.1 x 10^{-26} kg·m·s^{-1}

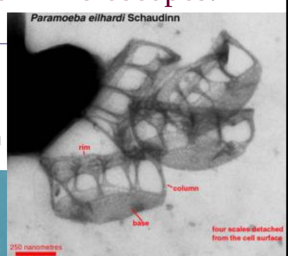
$$\lambda = \frac{h}{p} = \frac{6.62x10^{-34}}{9.1x10^{-26}} = 7.3x10^{-9} m$$

Wave properties of an electron

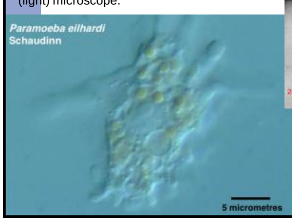
- The space between atoms in a crystal structure is on the same order of magnitude as the electron's de Broglie wavelength
- If a beam of electrons is directed at a crystal, then the electrons will scatter (diffract)
  - (basic principle behind the electron microscope)—(see http://www.bigelow.org/electron/)

Sample images from microscopes:

Right: Image of the amoeba shown below, but using an electron microscope—shows much more detail!



Below: Image of an amoeba using a traditional (light) microscope.



Both Images from http://www.bigelow.org/electron/

Curious observations…

- As seen in our optics unit, the intensity of the light does not affect the diffraction pattern that photons exhibit when passing through 2 slits, for example.
- Electrons, since they exhibit the same type of diffraction patterns, also have the same result: the intensity of the electron beam has nothing to do with the pattern seen when diffracted.
- SO: even if there is only one electron passing through a crystal, **it will still exhibit the diffraction pattern that is typically a result of interference!!**

(Crazy, huh? ☺)