Errors and Uncertainties

in measurements and in calculations

Types of Experimental Errors:

- Random Errors:
  - A result of variations in the performance of the instrument and/or the operator
  - Do NOT consistently occur throughout a lab
  - Some examples:
    - Vibrations or air currents when measuring mass
    - Inconsistent temperature (i.e. of the air) throughout a lab
    - Irregularities in object being measured (i.e. the wire is not the same thickness at all points along its length)
    - Human parallax error

- So what can be done about random errors?
  - Don’t rush through your measurements! Be careful!
  - Take as many trials as possible—the more trials you do, the less likely one odd result will impact your overall lab results
Types of Experimental Errors:

- **Systematic Errors:**
  - Errors that are inherent to the system or the measuring instrument
  - Results in a set of data to be centered around a value that is different than the accepted value
- **Some Examples:**
  - Non-calibrated (or poorly calibrated) measuring tools
  - A “zero offset” on a measuring tool, requiring a "zero correction"
  - Instrument parallax error

What can be done to reduce these?

- Unfortunately, nothing... HOWEVER:
- We can account for the systematic errors sometimes:
  - i.e. if there’s a zero offset, make sure all your data has been adjusted to account for that.
- Recognizing systematic errors will impact the size of your **absolute uncertainty** (more details soon 😊)

Are these “errors”?

- Misreading the scale on a triple-beam balance
- Incorrectly transferring data from your rough data table to the final, typed, version in your report
- Miscalculating results because you did not convert to the correct fundamental units
- Miscalculations because you use the wrong equation
Are these “errors”?

- NONE of these are experimental errors
- They are MISTAKES
- What’s the difference?
  - You need to check your work to make sure these mistakes don’t occur... ask questions if you need to (of your lab partner, me, etc.)
  - Do NOT put mistakes in your error discussion in the conclusion

Uncertainties in Measurement

- Limit of Reading:
  - Equal to the smallest graduation of the scale on an instrument
- Degree of Uncertainty:
  - Equal to half the limit of reading
  - Gives an indication of the precision of the reading

Uncertainties in Measurement

- Absolute Uncertainty:
  - The size of an error, including units
  - The SMALLEST the uncertainty can be is equal to the degree of uncertainty, however it is ALMOST ALWAYS BIGGER!
  - The absolute uncertainty can NOT be more precise than your measurement
  - The absolute uncertainty is ALWAYS reported to only 1 sig. fig.
  - Note: +/- is sometimes symbolized with Δ (Greek letter Delta)
Examples:

- Acceptable: 1.62 +/- 0.01 m
- NOT acceptable: 1.62 +/- 0.005 m

More uncertainties in measurement

- Relative (fractional) uncertainty:
  - Equal to the ratio of the absolute uncertainty to the measurement:
    $$\frac{\text{absolute uncertainty}}{\text{measurement}}$$
- Percentage uncertainty:
  - (fractional uncertainty) x 100 = %

Uncertainty Propagation

- When we perform calculations with measurements that have uncertainties, there is a certain amount of uncertainty in our calculated answer as well.
- Carrying our errors through the calculations is called “error propagation” or “uncertainty propagation”
Rule 1: Addition/Subtraction

- When you are adding or subtracting values in a calculation, the uncertainty in the calculated answer is equal to the sum of the absolute uncertainties for each measurement added:
  
  \[ 24.10 \pm 0.05 \text{ g} \]
  
  \[ + 13.05 \pm 0.02 \text{ g} \]
  
  \[ = 37.15 \pm 0.07 \text{ g} \]

Rule #2: Multiplying/Dividing

- When multiplying or dividing, the uncertainty in the calculated value is equal to the sum of the percentage uncertainties for each of the individual measurements:
  
  For example, let’s say we were to calculate the volume from the following measurements:

  \[(12.0 \pm 0.2 \text{ cm}) \times (23.1 \pm 0.2 \text{ cm}) \times (7.5 \pm 0.1 \text{ cm})\]

  - Step 1: determine the % uncertainty for each measurement
    
    \[
    \frac{0.2}{12.0} \times 100 = 1.7\% \\
    \frac{0.2}{23.1} \times 100 = 0.9\% \\
    \frac{0.1}{7.5} \times 100 = 1.3\% 
    \]

  - Step 2: Add all of the % uncertainties and round to 1-2 sig figs (usually a whole number, although if under 1 keep as a one s.f. decimal percentage):
    
    \[1.7\% + 0.9\% + 1.3\% = 3.9\% \approx 4\%\]
Step 3: convert the percentage uncertainty in your answer back to an absolute uncertainty:

\[(12.0 \pm 0.2 \text{ cm}) \times (23.1 \pm 0.2 \text{ cm}) \times (7.5 \pm 0.1 \text{ cm})\]
\[= 2100 \text{ cm}^3 \pm 4\%\]

\[0.04 \times 2100 \text{ cm}^3 = 84 \text{ cm}^3 = 100 \text{ cm}^3\]

(note: since the precision of the measurement and the uncertainty MUST BE THE SAME, always go with the least precise of the two when reporting your final answer.)

\[V = 2100 \pm 100 \text{ cm}^3\]

Rule #3 (special case for Averages)

Looking at the table, how would you calculate the average mass?

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Mass (±0.05 kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.95</td>
</tr>
<tr>
<td>2</td>
<td>3.92</td>
</tr>
<tr>
<td>3</td>
<td>4.00</td>
</tr>
<tr>
<td>4</td>
<td>4.08</td>
</tr>
<tr>
<td>5</td>
<td>3.98</td>
</tr>
</tbody>
</table>

\[m_{av} = \frac{\sum m}{n} = \frac{3.95 + 3.92 + 4.00 + 4.08 + 3.98}{5}\]

\[m_{av} = 3.986 \text{ kg} = 3.99 \text{ kg}\]

What about uncertainties?

Is the uncertainty STILL ±0.05 kg?

No! (Why not?)

Correct way to report uncertainty in average:

\[\text{Uncert.} = \frac{1}{2} \text{ (range)}\]

\[unc = \frac{1}{2} (4.08 - 3.92) = \frac{1}{2} (0.16) = 0.08 \text{ kg}\]

\[m_{av} = 3.99 \pm 0.08 \text{ kg}\]