Warm-up (11.16.15)

Madame Hooch throws a Quaffle straight up into the air to begin a Quidditch match. It was caught at its maximum height, 3.25 m.

1. What is the acceleration of the quaffle?
2. What is its velocity at its maximum height?
3. What was the velocity at which it was thrown?
4. How much time was it in the air between when it was released and when it was caught?
5. If the Chaser misses the Quaffle and it falls back to the height at which it was thrown, what is its velocity at that point?

Projectile Motion

Horizontally Launched Projectiles
Projectiles Launched at an Angle
Text Reference: pp. 39-43

Question 3.6a

Dropping the Ball I

From the same height (and at the same time), one ball is dropped and another ball is fired horizontally. Which one will hit the ground first?

a) the “dropped” ball
b) the “fired” ball
c) they both hit at the same time
d) it depends on how hard the ball was fired
e) it depends on the initial height
**Question 3.6b**  
**Dropping the Ball II**

In the previous problem, which ball has the greater velocity at ground level?

- a) the “dropped” ball
- b) the “fired” ball
- c) neither—they both have the same velocity on impact
- d) it depends on how hard the ball was thrown

**Question 3.4a**  
**Firing Balls I**

A small cart is rolling at constant velocity on a flat track. It fires a ball straight up into the air as it moves. After it is fired, what happens to the ball?

- a) it depends on how fast the cart is moving
- b) it falls behind the cart
- c) it falls in front of the cart
- d) it falls right back into the cart
- e) it remains at rest

**Question 3.4b**  
**Firing Balls II**

Now the cart is being pulled along a horizontal track by an external force (a weight hanging over the table edge) and accelerating. It fires a ball straight out of the cannon as it moves. After it is fired, what happens to the ball?

- a) it depends upon how much the track is tilted
- b) it falls behind the cart
- c) it falls in front of the cart
- d) it falls right back into the cart
- e) it remains at rest
The same small cart is now rolling down an inclined track and accelerating. It fires a ball straight out of the cannon as it moves. After it is fired, what happens to the ball?

a) It depends upon how much the track is tilted
b) It falls behind the cart
c) It falls in front of the cart
d) It falls right back into the cart
e) It remains at rest

Components of Motion--horizontal

- Horizontal motion is unaffected by vertical accelerations or vertical motion
- We will work with the assumption that air resistance is negligible, so...
- Horizontal velocity will remain CONSTANT throughout the flight of a projectile

Horizontally Launched: Problem Solving Strategy

<table>
<thead>
<tr>
<th>X-direction</th>
<th>Y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>v =</td>
<td>u =</td>
</tr>
<tr>
<td>s =</td>
<td>v =</td>
</tr>
<tr>
<td>t =</td>
<td>a =</td>
</tr>
<tr>
<td>s =</td>
<td>t =</td>
</tr>
</tbody>
</table>

- Horizontal direction:
  - v is constant
  - t = total time in air
  - s = horizontal displacement
    \[ s = vt \]
- Vertical direction:
  - u = 0 when launched horizontally
  - s = height
  - t = total time in air
  - a = - 9.81 m/s² (assume down is negative...)
  - Use Kinematic equations!
Practice Problem

A marble rolls across a horizontal table with a speed of 2.50 m·s\(^{-1}\). If the table is 1.12 m high, how far away from the edge of the table will the marble land?

<table>
<thead>
<tr>
<th>X-direction</th>
<th>Y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>v = 2.50 m·s(^{-1})</td>
<td>u = 0</td>
</tr>
<tr>
<td>s = ?</td>
<td>v =</td>
</tr>
<tr>
<td>t = ?</td>
<td>a = -9.81 m·s(^{-2})</td>
</tr>
<tr>
<td>s = 1.12 m</td>
<td></td>
</tr>
<tr>
<td>t = ?</td>
<td></td>
</tr>
</tbody>
</table>

Step 1: Solve for time in the air. Which direction gives you enough information to find the time?

Step 2: Calculate the horizontal displacement from the calculated time and the given velocity.

General Terms/Characteristics

- **Range**: the horizontal displacement of a projectile
- **Trajectory**: The path the projectile follows while in the air
- **Flight Time (or, Hang Time)**: The total amount of time the projectile is in the air.
- **Maximum Height**: The vertical displacement a projectile has reached when its vertical velocity is zero.
Velocity Components

- Projectile motion can only be analyzed by breaking the initial velocity into its horizontal and vertical components:

\[
\begin{align*}
v_x &= v \cos(\theta) \\
v_y &= v \sin(\theta)
\end{align*}
\]

- Just like with horizontal launches, the horizontal velocity will remain constant, while the vertical motion will be affected by gravity.

Problem Solving Technique:

- Just like with horizontally launched projectiles, you must separate all variables into the appropriate direction:

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>(v_x = v \cos(\theta))</td>
<td>(v_y = v \sin(\theta))</td>
</tr>
<tr>
<td>(d_x = \text{range})</td>
<td>(a = -9.81 \text{ m/s}^2)</td>
</tr>
<tr>
<td>(t = \text{flight time})</td>
<td>(d_y = \text{Max. height})</td>
</tr>
<tr>
<td></td>
<td>(t = \frac{1}{2} \text{flight time})</td>
</tr>
</tbody>
</table>

Example Problem #1:

- A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal. Find the ball's:
  - hang time
  - Range
  - maximum height
Example #2

- An arrow is shot at 30.0° above the horizontal. Its velocity is 49.0 m/s and it hits the target.
- What is the maximum height the arrow will attain?
- The target is at the same height from which the arrow was originally shot. How far away is it?

Example #3

- A soccer ball is kicked into the air at a velocity of 29.0 m/s and at an angle of 37.0°. How far away will it first hit the ground? (Assuming we ignore air resistance)

Range Equation

- The kinematic equations for the vertical and the horizontal directions can be combined mathematically in order to solve for the horizontal range WITHOUT KNOWING THE TIME IN THE AIR!

\[ R = \frac{2v_i^2 \sin \theta \cos \theta}{g} \]

- \( R \) = range (m)
- \( V_i \) = overall launch velocity (the total vector quantity) (m/s)
- \( g = 9.81 \text{ m/s}^2 \) → Note: this is NOT negative!
Example #3

A pitched ball is hit by a batter at a 45° angle and just clears the outfield fence, 98 m away. Assuming that the fence is at the same height as the pitch, find the velocity of the ball it left the bat.