Measurements and Uncertainties

Sig. Figs, Accuracy/Precision, and Uncertainty rules

Section 1: Significant Figures

Super important tool to use with measurements!

Significant Figures (sig. figs.)

- All digits in a measurement that are known for certain, plus the first estimated (uncertain) digit
- Sig figs give an indication of the degree of precision for a measurement and/or a calculation
- ONLY used when a number is (or is assumed to be) a measurement
- EXACT quantities do not have "sig figs"
  - Examples:
    - there are exactly 100 cm in 1 m
    - By definition, 1 inch = 2.54 cm
    - Fractions in equations (1/2 or 0.5 as a multiplier)

How many significant figures are in the following measurement:
3508.2 g

How many significant figures are in the following measurement:
3508.2 g

- 5
- Rules:
  - All non-zero values ARE significant
  - All zeros between non-zero digits ARE significant

How many significant figures are in the following measurement:
0.00065 s

How many significant figures are in the following measurement:
0.00065 s

- 2
- Rule:
  - For numbers LESS THAN 1:
    - Zeros directly after the decimal point are NOT significant
- The zeros at the beginning are simply placeholders
- You can rewrite the number with new units or in scientific notation, and the zeros will drop out.

How many significant figures are in the following measurement:
1500 g

How many significant figures are in the following measurement:
1500 g

- 2
- Rule:
  - All non-zero values ARE significant
  - If there is no decimal point, then the zeros at the end of a number (after the last non-zero digit) are insignificant.
  - These zeros are placeholders—we can rewrite the value with different units or in scientific notation and the zeros will drop out.

How many significant figures are in the following measurement:
0.007250 W

How many significant figures are in the following measurement:
0.007250 W

- **4**
- Rule:
  - A zero to the right of a decimal AND following a non-zero digit IS significant

The last zero is not a placeholder—it is showing the level of precision of the measurement.

The first 3 zeros are still placeholders, so not significant

How many significant figures are in the following measurement:
105.00 cm

How many significant figures are in the following measurement:
105.00 cm

- **5**

- Rules:
  - All non-zero values ARE significant
  - All zeros between non-zero digits ARE significant
  - A zero to the right of a decimal AND following a non-zero digit IS significant

What is the answer to the following calculation, written in proper sig. figs:

1.25 cm + 6.5 cm + 11.75 cm + 0.055 cm

What is the answer to the following calculation, written in proper sig. figs:

1.25 cm + 6.5 cm + 11.75 cm + 0.055 cm

$= 19.555 \approx \mathbf{19.6}\ \boldsymbol{cm}$

- Rule:
- When adding or subtracting:
  - Your answer must have the same degree of precision as the <u>**least precise measurement**</u>
  - (that means…go to the fewest number of decimal places, if there are decimal places in use)

What is the answer to the following calculation, written to proper sig figs:
25.50 m * 12.057 m * 0.095 m

What is the answer to the following calculation, written to proper sig figs:

25.50 m * 12.057 m * 0.095 m

$= 29.208\ m^3 \approx \mathbf{29}\ m^3$

- Rule:
- When multiplying and dividing:
  - The number of sig figs in the answer is equal to the **least number of sig figs** in any of the measurements used in the calculation

Scientific notation and sig figs

- Use Scientific notation when you need to specify how many zeros are significant
- i.e. Write 1500 N with 3 s.f.
  - The best way to do this is with scientific notation:
    1.50×10^3 N
- Write 10600 kg with 4 s.f.
  $1.060\ x\ 10^4\ kg$

Note on book problems:

- Most of the problems in your book will have values which look like they only have 1 s.f.
- **Assume that all digits in book problems *are* significant**
- i.e. if a problem says that an object has a mass of 100 kg, please treat that as 3 s.f.
- As long as you don't go overboard, don't worry about sig figs in your WebAssign problems—wait until the very end for any rounding!
- **We'll be a lot pickier in your labs and quizzes/tests!**

Section 2: Accuracy vs. Precision

Accuracy
Precision of a Measurement
Precision of a Data Set

Accuracy

- Accuracy of a measurement:
  - An indication of how close the measurement is to the accepted value
  - Percentage difference can be calculated to give a quantitative indication of a measurement's accuracy—the smaller the percentage difference, the greater the accuracy
  - Good accuracy is an indication of low systematic error.

Precision of a Measurement

- An indication of how "exactly" you can measure a piece of data
- More precise measurements are those that are measured to a smaller increment of a unit of measure (i.e. more decimal places)
  - Example: a thickness of wire measured with a meter stick will be precise to 0.01 cm; using a micrometer can increase the precision to 0.0001 cm
- ALWAYS use a measuring tool that will give you the most appropriate precision.
- Absolute uncertainty can be used to indicate the precision of your measurement

Precision of a Data Set

- an indication of the agreement among a number of measurements made in the same way (i.e. with the same measuring tool and procedure)
- The more consistent your results are, the higher the precision is
- High precision implies small amount of random error

Journal—Mini-Lab

- Measurements to the correct precision using an appropriate tool.
- Worksheet for this lab should be inserted in your journal.

Section 3: Types of Experimental Errors

Random Errors

Systematic Errors

T&T: Which of these are "experimental errors"? Explain.

1. Misreading the scale on a triple-beam balance
2. Incorrectly transferring data from your rough data table to the final, typed, version in your report
3. Miscalculating results because you did not convert to the correct fundamental units
4. Miscalculations because you use the wrong equation

Are these "errors"?

- NONE of these are experimental errors
- They are MISTAKES
- What's the difference?
  - You need to check your work to make sure these mistakes don't occur…ask questions if you need to (of your lab partner, me, etc.)
  - Do NOT put mistakes in your error discussion in the conclusion

Types of *Experimental* Errors:

- **Random Errors:**
  - A result of variations in the performance of the instrument and/or the operator
  - Do NOT consistently occur throughout a lab
- Some examples:
  - Vibrations or air currents when measuring mass
  - Inconsistent temperature (i.e. of the air) throughout a lab
  - Irregularities in object being measured (i.e. the wire is not the same thickness at all points along its length)
  - Reaction time when using a stopwatch

Types of *Experimental* Errors:

- So what can be done about random errors?
  - Don't rush through your measurements! Be careful!
  - Take as many trials as possible—the more trials you do, the less likely one odd result will impact your overall lab results

Types of *Experimental* Errors:

- **Systematic Errors:**
  - Errors that are inherent to the system or the measuring instrument
  - Results in a set of data to be centered around a value that is different than the accepted value
- Some Examples:
  - Non-calibrated (or poorly calibrated) measuring tools
  - A "zero offset" on a measuring tool, requiring a "zero correction"
  - A warped ruler—results in non-symmetrical divisions

Types of ***Experimental*** Errors:

- What can be done to reduce these?
  - Unfortunately, often there is nothing you can do…unless you repeat the experiment with another piece of equipment
  - We can account for the systematic errors sometimes:
    - i.e. if there's a zero offset, make sure all your data has been adjusted to account for that.
  - Recognizing systematic errors will impact the size of your absolute uncertainty (more details soon)

Uncertainties in Measurement

- **Limit of Reading**:
  - Equal to the smallest graduation of the scale on an instrument
- **Degree of Uncertainty**:
  - Equal to half the limit of reading (for non-digital measuring tools)
  - Gives an indication of the precision of the reading

Uncertainties in Measurement

- **Absolute Uncertainty**:
  - The size of an error, including units
  - The SMALLEST the uncertainty can be is equal to the degree of uncertainty, however it is ALMOST ALWAYS **BIGGER**!
- The absolute uncertainty can NOT be more precise than your measurement
- The absolute uncertainty is ALWAYS reported to only 1 sig. fig.
- Note: +/- is sometimes symbolized with Δ (Greek letter Delta)

Examples:

- Acceptable:
  1.62 +/- 0.01 m
- NOT acceptable:
  1.62 +/- 0.005 m

More uncertainties in measurement

- Relative (fractional) uncertainty:
  - Equal to the ratio of the absolute uncertainty to the measurement:

$$\frac{\text{absolute uncertainty}}{\textit{measurement}}$$

- Percentage uncertainty:
  - (fractional uncertainty) x 100 = %

Uncertainty Propagation

- When we perform calculations with measurements that have uncertainties, there is a certain amount of uncertainty in our calculated answer as well.
- Carrying our errors through the calculations is called “error propagation” or “uncertainty propagation”

Think about this…

- If you were measuring the width of a hallway using only a single meter stick, how would you handle the uncertainty of your measurement, knowing that the distance to measure is more than 2 m?

Rule 1: Addition/Subtraction

- When you are adding or subtracting values in a calculation, the uncertainty in the calculated answer is equal to the sum of the absolute uncertainties for each measurement added:

24.10 ± 0.05 g
+ 13.05 ± 0.02 g
= **37.15 ± 0.07 g**

24.10 ± 0.05 g
+ 13.05 ± 0.05 g
= **37.15 ± 0.10 g**

→37.2 ± 0.1 g

Rule #2: Multiplying/Dividing

- When multiplying or dividing, the uncertainty in the calculated value is equal to the sum of the **percentage uncertainties** for each of the individual measurements:

For example, let's say we were to calculate the volume from the following measurements:

(12.0 ± 0.2 cm)*(23.1 ± 0.2 cm)*(7.5 ± 0.1 cm)

- Step 1: determine the % uncertainty for each measurement→

$$\frac{0.2}{12.0}\times 100 = 1.7\%$$

$$\frac{0.2}{23.1}\times 100 = 0.9\%$$

$$\frac{0.1}{7.5}\times 100 = 1.3\%$$

- Step 2: Add all of the % uncertainties and round to 1-2 sig figs *(usually a whole number, although if under 1 keep as a one s.f. decimal percentage:*

$$1.7\% + 0.9\% + 1.3\%$$

$$= 3.9\% \approx 4\%$$

- Step 3: convert the percentage uncertainty in your answer back to an absolute uncertainty:

(12.0 ± 0.2 cm)*(23.1 ± 0.2 cm)*(7.5 ± 0.1 cm)

= 2100 cm^3 ± 4%

0.04 x 2100 cm^3 = 84 cm^3 ≈ 100cm^3

(*note: since the precision of the measurement and the uncertainty MUST BE THE SAME, always go with the least precise of the two when reporting your final answer.)*

V = 2100 ± 100 cm^3

Rule #3 (special case for Averages)

- Looking at the table, how would you calculate the average mass?

$$m_{av} = \frac{\Sigma m}{n} =$$

$$m_{av} = \frac{3.95 + 3.92 + 4.00 + 4.08 + 3.98}{5}$$

$$m_{av} = 3.986kg = 3.99kg$$

Trial #	Mass (±0.02 kg)
1	3.95
2	3.92
3	4.00
4	4.08
5	3.98

- What about uncertainties?

Rule #3 (special case for Averages)

- Is the uncertainty STILL ±0.02 kg?
- NO! (Why not?)
- Correct way to report uncertainty in average:
  Uncert. = ½ (range)

Trial #	Mass (±0.02 kg)
1	3.95
2	3.92
3	4.00
4	4.08
5	3.98

$$unc = \tfrac{1}{2}(4.08 - 3.92) = \tfrac{1}{2}(0.16) = 0.08kg$$

$$m_{av} = 3.99 \pm 0.08kg$$

Uncertainties in Graphs

- Error Bars:
  - Bars drawn horizontally or vertically around a data point that indicate the possible range of values for that data point's measurement
  - The distance to either side of the data point is equivalent to the magnitude of the absolute uncertainty for that point.