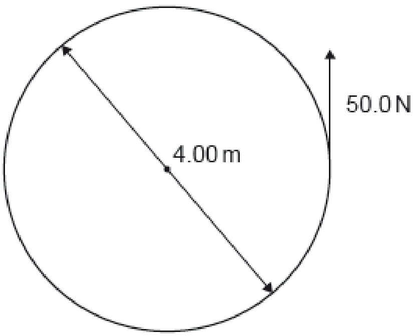
Rotational Dynamics Practice Problems *[59 marks]*

A constant force of 50.0 N is applied tangentially to the outer edge of a merry-go-round. The following diagram shows the view from above.



The merry-go-round has a moment of inertia of 450 kg m^2 about a vertical axis. The merry-go-round has a diameter of 4.00 m.

1a. Show that the angular acceleration of the merry-go-round is 0.2 rad s^{-2}. *[2 marks]*

Markscheme

Γ «= *Fr* = 50 × 2» = 100 «Nm»

α «$= \frac{\Gamma}{I} = \frac{100}{450}$» =0.22 «rad$s^{-2}$»

Final value to at least 2 sig figs, ***OR*** *clear working with substitution required for mark.*

[2 marks]

The merry-go-round starts from rest and the force is applied for one complete revolution.

1b. Calculate, for the merry-go-round after one revolution, the angular speed. *[1 mark]*

Markscheme

«$\omega_t^2 - \omega_0^2 = 2\alpha\Delta\theta$»

«$\omega_t^2 - 0 = 2 \times 0.22 \times 2\pi$»

$\omega_t = 1.7$ «rads^{-1}»

Accept BCA, values in the range: 1.57 to 1.70.

[1 mark]

1c. Calculate, for the merry-go-round after one revolution, the angular momentum. *[1 mark]*

Markscheme

«$L = I\omega = 450 \times 1.66$»

= 750 «kgm^2 rads^{-1}»

Accept BCA, values in the range: 710 to 780.

[1 mark]

A child of mass 30.0 kg is now placed onto the edge of the merry-go-round. No external torque acts on the system.

1d. Calculate the new angular speed of the rotating system. *[2 marks]*

Markscheme

«$I = 450 + mr^2$»

I «$= 450 + 30 \times 2^2$» $= 570$ «kgm^2»

«$L = 570 \times \omega = 747$»

$\omega = 1.3$ «$rads^{-1}$»

Watch for ECF from (a) and (b).

Accept BCA, values in the range: 1.25 to 1.35.

[2 marks]

The child now moves towards the centre.

1e. Explain why the angular speed will increase. *[2 marks]*

Markscheme

moment of inertia will decrease

angular momentum will be constant «as the system is isolated»

«so the angular speed will increase»

[2 marks]

1f. Calculate the work done by the child in moving from the edge to the centre. *[2 marks]*

Markscheme

$\omega_t = 1.66$ from bi ***AND*** $W = \Delta E_k$

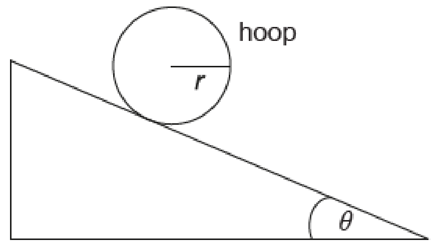
$W = \frac{1}{2} \times 450 \times 1.66^2 - \frac{1}{2} \times 570 \times 1.31^2 = 131$ «J»

ECF from 8bi

Accept BCA, value depends on the answers in previous questions.

[2 marks]

A hoop of mass *m*, radius *r* and moment of inertia mr^2 rests on a rough plane inclined at an angle θ to the horizontal. It is released so that the hoop gains linear and angular acceleration by rolling, without slipping, down the plane.



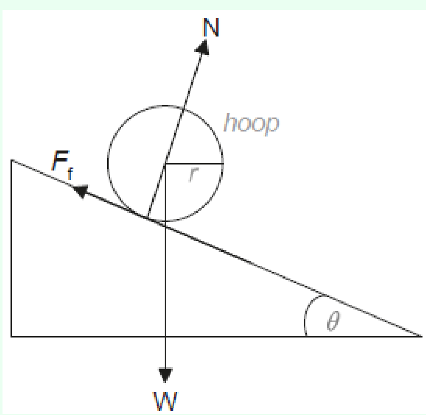
2a. On the diagram, draw and label the forces acting on the hoop. *[2 marks]*

Markscheme

weight, normal reaction and friction in correct direction

correct points of application for at least two correct forces

Labelled on diagram.



Allow different wording and symbols

Ignore relative lengths

2b. Show that the linear acceleration *a* of the hoop is given by the equation shown. *[4 marks]*

$a = \frac{g\times\sin q}{2}$

Markscheme

ALTERNATIVE 1

$ma = mg \sin \theta - F_f$

$I\alpha = F_f \times r$

OR

$mr\alpha = F_f$

$\alpha = \frac{a}{r}$

$ma = mg \sin \theta - mr\frac{a}{r} \rightarrow 2a = g \sin \theta$

Can be in any order

No mark for re-writing given answer

Accept answers using the parallel axis theorem (with $I = 2mr^2$) only if clear and explicit mention that the only torque is from the weight

Answer given look for correct working

ALTERNATIVE 2

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

substituting $\omega = \frac{v}{r}$ «giving $v = \sqrt{gh}$»

correct use of a kinematic equation

use of trigonometry to relate displacement and height «$s = h \sin \theta$»

For alternative 2, MP3 and MP4 can only be awarded if the previous marking points are present

2c. Calculate the acceleration of the hoop when $\theta = 20°$. Assume that the hoop continues to roll without slipping. *[1 mark]*

Markscheme

1.68 «ms^{-2}»

2d. State the relationship between the force of friction and the angle of the incline. *[2 marks]*

Markscheme

ALTERNATIVE 1

$N = mg\cos\theta$

$F_f \le \mu mg\cos\theta$

ALTERNATIVE 2

$F_f = ma$ «from 7(b)»

so $F_f = \frac{mg\sin\theta}{2}$

2e. The angle of the incline is slowly increased from zero. Determine the angle, in terms of the coefficient of friction, at which the hoop will begin to slip. *[3 marks]*

Markscheme

$F_f = \mu mg\cos\theta$

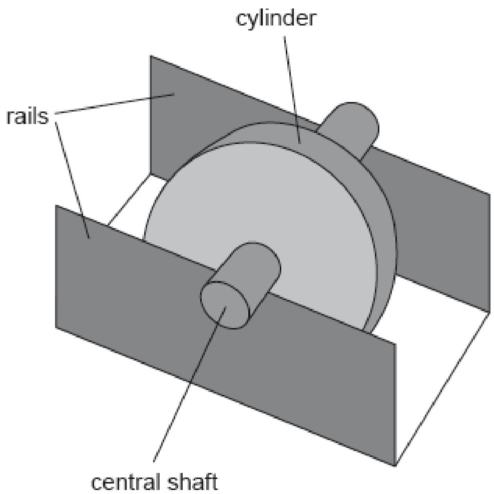
$\frac{mg\sin\theta}{2} = mg\sin\theta - \mu mg\cos\theta$

OR

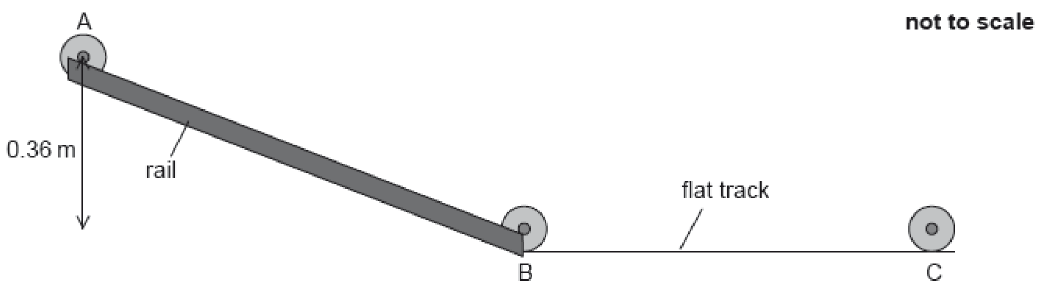
$mg\frac{\sin\theta}{2} = \mu mg\cos\theta$

algebraic manipulation to reach $\tan\theta = 2\mu$

A wheel of mass 0.25 kg consists of a cylinder mounted on a central shaft. The shaft has a radius of 1.2 cm and the cylinder has a radius of 4.0 cm. The shaft rests on two rails with the cylinder able to spin freely between the rails.



The stationary wheel is released from rest and rolls down a slope with the shaft rolling on the rails without slipping from point A to point B.



3a. The moment of inertia of the wheel is 1.3×10^{-4} kg m^2. Outline what is meant by the moment of inertia. *[1 mark]*

Markscheme

an object's resistance to change in rotational motion

OR

equivalent of mass in rotational equations

OWTTE

[1 mark]

3b. In moving from point A to point B, the centre of mass of the wheel falls through a vertical distance of 0.36 m. Show that the translational speed of the wheel is about 1 m s^{-1} after its displacement. *[3 marks]*

Markscheme

ΔKE + Δrotational KE = ΔGPE

OR

$\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mgh$

$\frac{1}{2} \times 0.250 \times v^2 + \frac{1}{2} \times 1.3 \times 10^{-4} \times \frac{v^2}{1.44\times10^{-4}} = 0.250 \times 9.81 \times 0.36$

$v = 1.2$ «m s^{-1}»

[3 marks]

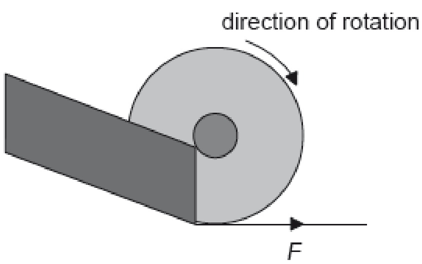
3c. Determine the angular velocity of the wheel at B. *[1 mark]*

Markscheme

ω «$= \frac{1.2}{0.012}$» = 100 «rad s^{-1}»

[1 mark]

The wheel leaves the rails at point B and travels along the flat track to point C. For a short time the wheel slips and a frictional force *F* exists on the edge of the wheel as shown.



3d. Describe the effect of *F* on the linear speed of the wheel. *[2 marks]*

Markscheme

force in direction of motion

so linear speed increases

[2 marks]

3e. Describe the effect of *F* on the angular speed of the wheel. *[2 marks]*

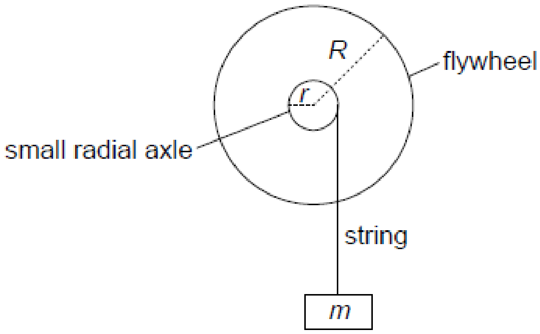
Markscheme

force gives rise to anticlockwise/opposing torque on wheel ✓ so angular speed decreases ✓

OWTTE

[2 marks]

A flywheel consists of a solid cylinder, with a small radial axle protruding from its centre.



The following data are available for the flywheel.

Flywheel mass *M*	= 1.22 kg
Small axle radius *r*	= 60.0 mm
Flywheel radius *R*	= 240 mm
Moment of inertia	= 0.5 MR^2

An object of mass *m* is connected to the axle by a light string and allowed to fall vertically from rest, exerting a torque on the flywheel.

4a. The velocity of the falling object is 1.89 m s^{-1} at 3.98 s. Calculate the average angular acceleration of the flywheel. *[2 marks]*

Markscheme

ALTERNATIVE 1

$\omega_{\text{final}} = \frac{v}{r} = 31.5$ «rad s^{-1}»

«$\omega = \omega_o + \alpha t$ so» $\alpha = \frac{\omega}{t} = \frac{31.5}{3.98} = 7.91$ «rad s^{-2}»

ALTERNATIVE 2

$a = \frac{1.89}{3.98} = 0.4749$ «m s^{-2}»

$\alpha = \frac{a}{r} = \frac{0.4749}{0.060} = 7.91$ «rad s^{-2}»

*Award **[1 max]** for r = 0.24 mm used giving α = 1.98 «rad s^{-2}».*

4b. Show that the torque acting on the flywheel is about 0.3 Nm. *[2 marks]*

Markscheme

$\Gamma = \frac{1}{2}\mathrm{MR}^2\alpha = \frac{1}{2} \times 1.22 \times 0.240^2 \times 7.91$

= 0.278 «Nm»

At least two significant figures required for MP2, as question is a "Show".

4c. (i) Calculate the tension in the string. *[4 marks]*

(ii) Determine the mass *m* of the falling object.

Markscheme

i

$F_T = \frac{\Gamma}{r}$

$F_T = 4.63$ «N»

Allow 5 «N» if Γ= 0.3 Nm is used.

ii

$F_T = mg - ma$ so $m = \frac{4.63}{9.81 - 0.475}$

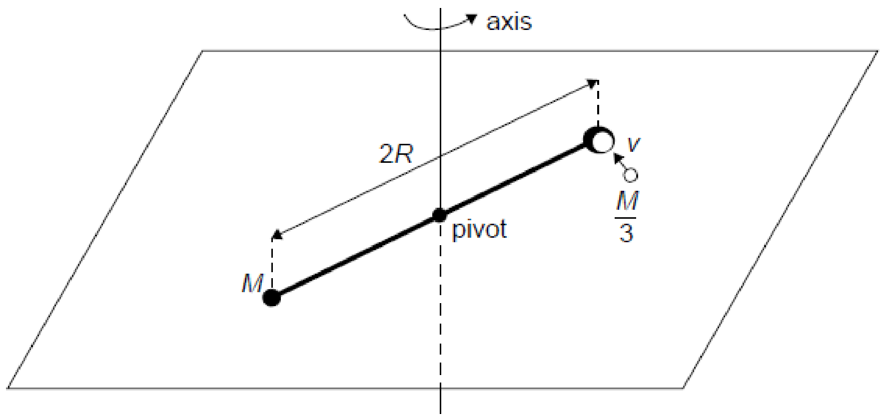
m = 0.496 «kg»

Allow ECF

A horizontal rigid bar of length 2*R* is pivoted at its centre. The bar is free to rotate in a horizontal plane about a vertical axis through the pivot. A point particle of mass *M* is attached to one end of the bar and a container is attached to the other end of the bar.

A point particle of mass $\frac{M}{3}$ moving with speed *v* at right angles to the rod collides with the container and gets stuck in the container. The system then starts to rotate about the vertical axis.

The mass of the rod and the container can be neglected.



5a. Write down an expression, in terms of *M*, *v* and *R*, for the angular momentum of the system about the vertical axis just before the collision. *[1 mark]*

Markscheme

$\frac{M}{3}vR$

[1 mark]

5b. Just after the collision the system begins to rotate about the vertical axis with angular velocity *ω*. Show that the angular momentum of the system is equal to $\frac{4}{3}MR^2\omega$. *[1 mark]*

Markscheme

evidence of use of: $L = I\omega = \left(MR^2 + \frac{M}{3}R^2\right)\omega$

[1 mark]

5c. Hence, show that $\omega = \frac{v}{4R}$. *[1 mark]*

Markscheme

evidence of use of conservation of angular momentum, $\frac{MvR}{3} = \frac{4}{3}MR^2\omega$

«rearranging to get $\omega = \frac{v}{4R}$»

[1 mark]

5d. Determine in terms of M and v the energy lost during the collision. *[3 marks]*

Markscheme

initial KE = $\frac{Mv^2}{6}$

final KE = $\frac{Mv^2}{24}$

energy loss = $\frac{Mv^2}{8}$

[3 marks]

A torque of 0.010 N m brings the system to rest after a number of revolutions. For this case R = 0.50 m, M = 0.70 kg and v = 2.1 m s^{-1}.

5e. Show that the angular deceleration of the system is 0.043 rad s^{-2}. *[1 mark]*

Markscheme

α «= $\frac{3}{4}\frac{\Gamma}{MR^2}$» = $\frac{3}{4}\frac{0.01}{0.7\times0.5^2}$

«to give α = 0.04286 rad s^{-2}»

*Working **OR** answer to at least 3 SF must be shown*

[1 mark]

5f. Calculate the number of revolutions made by the system before it comes to rest. *[3 marks]*

Markscheme

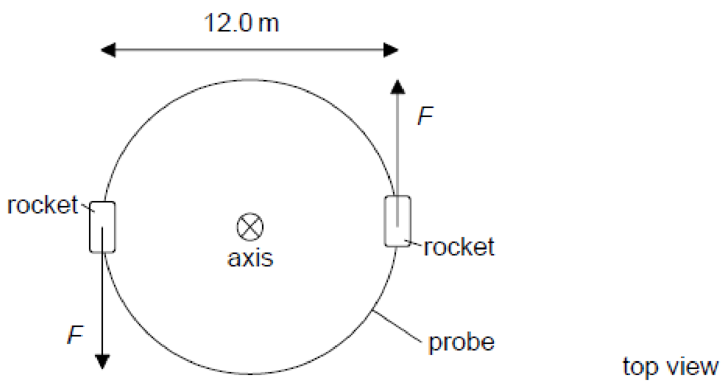
$\theta = \frac{\omega_i^2}{2\alpha}$ «from $\omega_f^2 = \omega_i^2 + 2\alpha\theta$»

θ «$= \frac{v^2}{32R^2\alpha} = \frac{2.1^2}{32 \times 0.5^2 \times 0.043}$» = 12.8 ***OR*** 12.9 «rad»

number of rotations «$= \frac{12.9}{2\pi}$» = 2.0 revolutions

[3 marks]

A cylindrical space probe of mass 8.00 x 10^2 kg and diameter 12.0 m is at rest in outer space.



Rockets at opposite points on the probe are fired so that the probe rotates about its axis. Each rocket produces a force F = 9.60 x 10^3 N. The moment of inertia of the probe about its axis is 1.44 x 10^4 kg m^2.

6a. Deduce the linear acceleration of the centre of mass of the probe. *[1 mark]*

Markscheme

zero

[1 mark]

6b. Calculate the resultant torque about the axis of the probe. *[2 marks]*

Markscheme

the torque of each force is 9.60 x 10^3 x 6.0 = 5.76 x 10^4 «Nm»

so the net torque is 2 x 5.76 x 10^4 = 1.15 x 10^5 «Nm»

Allow a one-step solution.

[2 marks]

6c. The forces act for 2.00 s. Show that the final angular speed of the probe is about 16 rad s^{-1}. *[2 marks]*

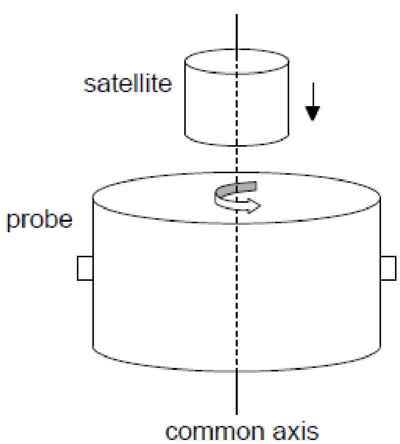
Markscheme

the angular acceleration is given by $\frac{1.15\times10^5}{1.44\times10^4}$ «= 8.0 s^{-2}»

ω = «αt = 8.0 x 2.00 =» 16 «s^{-1}»

[2 marks]

The diagram shows a satellite approaching the rotating probe with negligibly small speed. The satellite is not rotating initially, but after linking to the probe they both rotate together.



The moment of inertia of the satellite about its axis is 4.80 x 10^3 kg m^2. The axes of the probe and of the satellite are the same.

6d. Determine the final angular speed of the probe–satellite system. *[2 marks]*

Markscheme

$1.44 \times 10^4 \times 16.0 = (1.44 \times 10^4 + 4.80 \times 10^3) \times \omega$

$\omega = 12.0$ «s^{-1}»

Allow ECF from (b).

[2 marks]

6e. Calculate the loss of rotational kinetic energy due to the linking of the probe with the satellite. *[3 marks]*

Markscheme

initial KE $\frac{1}{2} \times 1.44 \times 10^4 \times 16.0^2 = 1.843 \times 10^6$ «J»

final KE $\frac{1}{2} \times \left(1.44 \times 10^4 + 4.80 \times 10^3\right) \times 12.0^2 = 1.382 \times 10^6$ «J»

loss of KE = 4.6×10^5 «J»

Allow ECF from part (c)(i).

[3 marks]