Rotational Motion *[59 marks]*

1a. *[2 marks]*

Markscheme

Γ «= *Fr* = 50 × 2» = 100 «Nm»

α « $= \frac{\Gamma}{I} = \frac{100}{450}$ » =0.22 «rads^{-2}»

*Final value to at least 2 sig figs, **OR** clear working with substitution required for mark.*

[2 marks]

1b. *[1 mark]*

Markscheme

«$\omega_t^2 - \omega_0^2 = 2\alpha\Delta\theta$»

«$\omega_t^2 - 0 = 2 \times 0.22 \times 2\pi$»

$\omega_t = 1.7$ «rads^{-1}»

Accept BCA, values in the range: 1.57 to 1.70.

[1 mark]

1c. *[1 mark]*

Markscheme

«$L = I\omega = 450 \times 1.66$»

= 750 «kgm^2 rads^{-1}»

Accept BCA, values in the range: 710 to 780.

[1 mark]

1d. *[2 marks]*

Markscheme

«$I = 450 + mr^2$»

I «= 450 + 30 × 2^2» = 570 «kgm^2»

«$L = 570 \times \omega = 747$»

ω = 1.3 «rads^{-1}»

Watch for ECF from (a) and (b).

Accept BCA, values in the range: 1.25 to 1.35.

[2 marks]

1e. *[2 marks]*

Markscheme

moment of inertia will decrease

angular momentum will be constant «as the system is isolated»

«so the angular speed will increase»

[2 marks]

1f. *[2 marks]*

Markscheme

ω_t = 1.66 from bi ***AND*** $W = \Delta E_k$

$W = \frac{1}{2} \times 450 \times 1.66^2 - \frac{1}{2} \times 570 \times 1.31^2 = 131$ «J»

ECF from 8bi

Accept BCA, value depends on the answers in previous questions.

[2 marks]

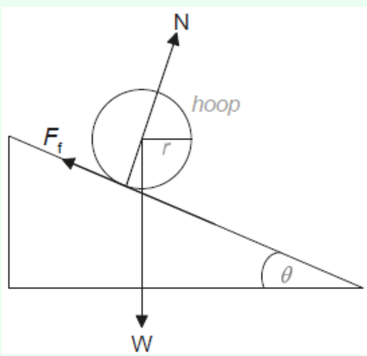
2a. *[2 marks]*

Markscheme

weight, normal reaction and friction in correct direction

correct points of application for at least two correct forces

Labelled on diagram.



Allow different wording and symbols

Ignore relative lengths

2b. *[4 marks]*

Markscheme

ALTERNATIVE 1

$ma = mg \sin \theta - F_f$

$I\alpha = F_f \times r$

OR

$mr\alpha = F_f$

$\alpha = \frac{a}{r}$

$ma = mg \sin \theta - mr \frac{a}{r} \rightarrow 2a = g \sin \theta$

Can be in any order

No mark for re-writing given answer

Accept answers using the parallel axis theorem (with $I = 2mr^2$) only if clear and explicit mention that the only torque is from the weight

Answer given look for correct working

ALTERNATIVE 2

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

substituting $\omega = \frac{v}{r}$ «giving $v = \sqrt{gh}$»

correct use of a kinematic equation

use of trigonometry to relate displacement and height « $s = h \sin \theta$»

For alternative 2, MP3 and MP4 can only be awarded if the previous marking points are present

2c. *[1 mark]*

Markscheme

1.68 «ms^{-2}»

2d. *[2 marks]*

Markscheme

ALTERNATIVE 1

$N = mg \cos \theta$

$F_f \leq \mu mg \cos \theta$

ALTERNATIVE 2

$F_f = ma$ «from 7(b)»

so $F_f = \frac{mg\sin\theta}{2}$

2e. *[3 marks]*

Markscheme

$F_f = \mu mg \cos \theta$

$\frac{mg\sin\theta}{2} = mg \sin \theta - \mu mg \cos \theta$

OR

$mg \frac{\sin\theta}{2} = \mu mg \cos \theta$

algebraic manipulation to reach $\tan \theta = 2\mu$

3a. *[1 mark]*

Markscheme

an object's resistance to change in rotational motion

OR

equivalent of mass in rotational equations

OWTTE

[1 mark]

3b. *[3 marks]*

Markscheme

ΔKE + Δrotational KE = ΔGPE

OR

$\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mgh$

$\frac{1}{2} \times 0.250 \times v^2 + \frac{1}{2} \times 1.3 \times 10^{-4} \times \frac{v^2}{1.44 \times 10^{-4}} = 0.250 \times 9.81 \times 0.36$

v = 1.2 «m s^{-1}»

[3 marks]

3c. *[1 mark]*

Markscheme

ω «= $\frac{1.2}{0.012}$» = 100 «rad s^{-1}»

[1 mark]

3d. *[2 marks]*

Markscheme

force in direction of motion

so linear speed increases

[2 marks]

3e. *[2 marks]*

Markscheme

force gives rise to anticlockwise/opposing torque on wheel ✓ so angular speed decreases ✓

OWTTE

[2 marks]

4a. *[2 marks]*

Markscheme

ALTERNATIVE 1

$\omega_{\text{final}} = \frac{v}{r} = 31.5$ «rad s^{-1}»

«$\omega = \omega_o + \alpha t$ so» $\alpha = \frac{\omega}{t} = \frac{31.5}{3.98} = 7.91$ «rad s^{-2}»

ALTERNATIVE 2

$a = \frac{1.89}{3.98} = 0.4749$ «m s^{-2}»

$\alpha = \frac{a}{r} = \frac{0.4749}{0.060} = 7.91$ «rad s^{-2}»

Award ***[1 max]*** *for r = 0.24 mm used giving*
α = 1.98 «rad s^{-2}».

4b. *[2 marks]*

Markscheme

$\Gamma = \frac{1}{2}\mathrm{MR}^2\alpha = \frac{1}{2} \times 1.22 \times 0.240^2 \times 7.91$

= 0.278 «Nm»

At least two significant figures required for MP2, as question is a "Show".

4c. *[4 marks]*

Markscheme

i

$F_T = \frac{\Gamma}{r}$

$F_T = 4.63$ «N»

Allow 5 «N» if Γ= 0.3 Nm is used.

ii

$F_T = mg - ma$ so $m = \frac{4.63}{9.81 - 0.475}$

m = 0.496 «kg»

Allow ECF

5a. *[1 mark]*

Markscheme

$\frac{M}{3}vR$

[1 mark]

5b. *[1 mark]*

Markscheme

evidence of use of: $L = I\omega = \left(MR^2 + \frac{M}{3}R^2\right)\omega$

[1 mark]

5c. *[1 mark]*

Markscheme

evidence of use of conservation of angular momentum, $\frac{MvR}{3} = \frac{4}{3}MR^2\omega$

«rearranging to get $\omega = \frac{v}{4R}$»

[1 mark]

5d. *[3 marks]*

Markscheme

initial KE = $\frac{Mv^2}{6}$

final KE = $\frac{Mv^2}{24}$

energy loss = $\frac{Mv^2}{8}$

[3 marks]

5e. *[1 mark]*

Markscheme

α «= $\frac{3}{4}\frac{\Gamma}{MR^2}$» = $\frac{3}{4}\frac{0.01}{0.7\times0.5^2}$

«to give α = 0.04286 rad s^{-2}»

*Working **OR** answer to at least 3 SF must be shown*

[1 mark]

5f. *[3 marks]*

Markscheme

$\theta = \frac{\omega_i^2}{2\alpha}$ «from $\omega_f^2 = \omega_i^2 + 2\alpha\theta$»

θ «$= \frac{v^2}{32R^2\alpha} = \frac{2.1^2}{32\times0.5^2\times0.043}$» = 12.8 ***OR*** 12.9 «rad»

number of rotations «= $\frac{12.9}{2\pi}$» = 2.0 revolutions

[3 marks]

6a. *[1 mark]*

Markscheme

zero

[1 mark]

6b. *[2 marks]*

Markscheme

the torque of each force is $9.60 \times 10^3 \times 6.0 = 5.76 \times 10^4$ «Nm»

so the net torque is $2 \times 5.76 \times 10^4 = 1.15 \times 10^5$ «Nm»

Allow a one-step solution.

[2 marks]

6c. *[2 marks]*

Markscheme

the angular acceleration is given by $\frac{1.15 \times 10^5}{1.44 \times 10^4}$ «= 8.0 s^{-2}»

ω = «αt = 8.0 x 2.00 =» 16 «s^{-1}»

[2 marks]

6d. *[2 marks]*

Markscheme

$1.44 \times 10^4 \times 16.0 = (1.44 \times 10^4 + 4.80 \times 10^3) \times \omega$

$\omega = 12.0$ «s^{-1}»

Allow ECF from (b).

[2 marks]

6e. *[3 marks]*

Markscheme

initial KE $\frac{1}{2} \times 1.44 \times 10^4 \times 16.0^2 = 1.843 \times 10^6$ «J»

final KE $\frac{1}{2} \times \left(1.44 \times 10^4 + 4.80 \times 10^3\right) \times 12.0^2 = 1.382 \times 10^6$ «J»

loss of KE = 4.6×10^5 «J»

Allow ECF from part (c)(i).

[3 marks]