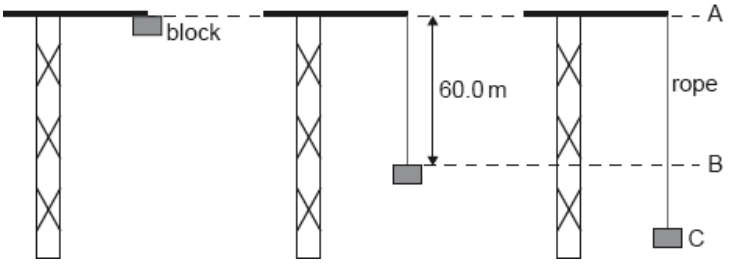
SHM practice *[46 marks]*

An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

1a. At position B the rope starts to extend. Calculate the speed of the block at position B. *[2 marks]*

Markscheme

use of conservation of energy

OR

$v^2 = u^2 + 2as$

$v = «\sqrt{2 \times 60.0 \times 9.81}» = 34.3$ «ms^{-1}»

[2 marks]

At position C the speed of the block reaches zero. The time taken for the block to fall between B and C is 0.759 s. The mass of the block is 80.0 kg.

1b. Determine the magnitude of the average resultant force acting on the block between B and C. *[2 marks]*

Markscheme

use of impulse $F_{ave} \times \Delta t = \Delta p$

OR

use of $F = ma$ with average acceleration

OR

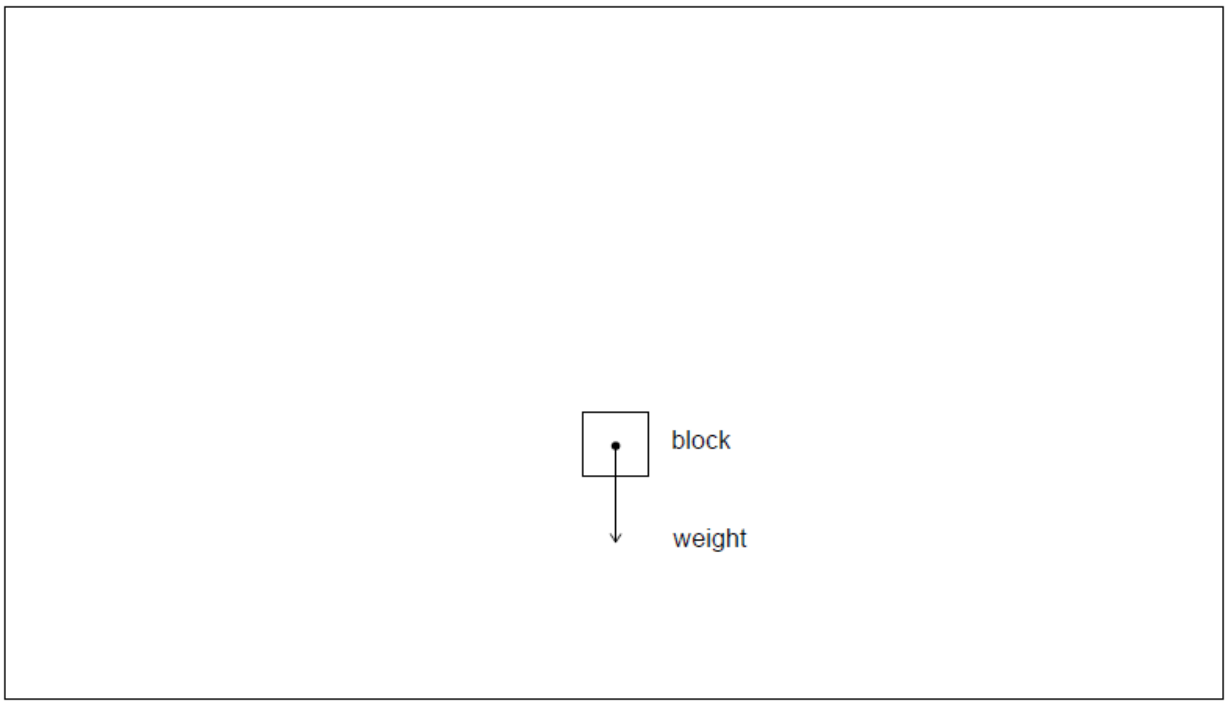
$F = \frac{80.0 \times 34.3}{0.759}$

3620«N»

Allow ECF from (a).

[2 marks]

1c. Sketch on the diagram the average resultant force acting on the block between B and C. The arrow on the diagram represents the weight of the block. *[2 marks]*



Markscheme

upwards

clearly longer than weight

For second marking point allow ECF from (b)(i) providing line is upwards.

[2 marks]

1d. Calculate the magnitude of the average force exerted by the rope on the block between B and C. *[2 marks]*

Markscheme

3620 + 80.0 × 9.81

4400 «N»

Allow ECF from (b)(i).

[2 marks]

For the rope and block, describe the energy changes that take place

1e. between A and B. *[1 mark]*

Markscheme

(loss in) gravitational potential energy (of block) into kinetic energy (of block)

Must see names of energy (gravitational potential energy and kinetic energy) – Allow for reasonable variations of terminology (eg energy of motion for KE).

[1 mark]

1f. between B and C. *[1 mark]*

Markscheme

(loss in) gravitational potential and kinetic energy of block into elastic potential energy of rope

See note for 1(c)(i) for naming convention.

Must see either the block or the rope (or both) mentioned in connection with the appropriate energies.

[1 mark]

1g. The length reached by the rope at C is 77.4 m. Suggest how energy considerations could be used to determine the elastic constant of the rope. *[2 marks]*

Markscheme

k can be determined using EPE = $\frac{1}{2}kx^2$

correct statement or equation showing

GPE at A = EPE at C

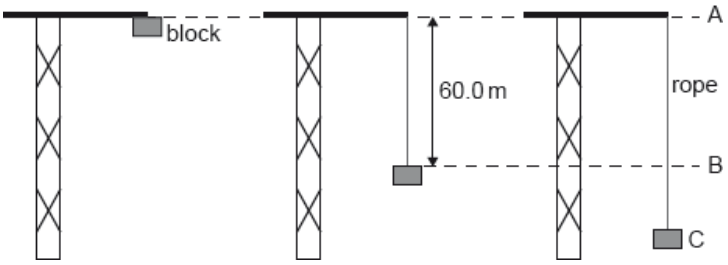
OR

(GPE + KE) at B = EPE at C

Candidate must clearly indicate the energy associated with either position A or B for MP2.

[2 marks]

An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

In another test, the block hangs in equilibrium at the end of the same elastic rope. The elastic constant of the rope is 400 Nm^{-1}. The block is pulled 3.50 m vertically below the equilibrium position and is then released from rest.

1h. Calculate the time taken for the block to return to the equilibrium position for the first time. *[2 marks]*

Markscheme

$T = 2\pi\sqrt{\frac{80.0}{400}} = 2.81$ «s»

time $= \frac{T}{4} = 0.702$ «s»

*Award **[0]** for kinematic solutions that assume a constant acceleration.*

[2 marks]

1i. Calculate the speed of the block as it passes the equilibrium position. *[2 marks]*

Markscheme

ALTERNATIVE 1

$\omega = \frac{2\pi}{2.81}$ = 2.24 «rad s^{-1}»

v = 2.24 × 3.50 = 7.84 «ms^{-1}»

ALTERNATIVE 2

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ ***OR*** $\frac{1}{2}400 \times 3.5^2 = \frac{1}{2}80v^2$

v = 7.84 «ms^{-1}»

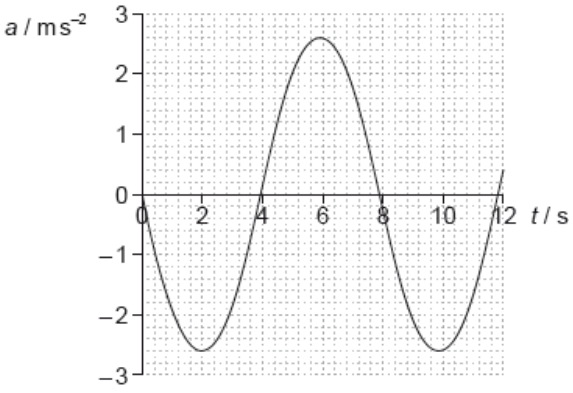
*Award **[0]** for kinematic solutions that assume a constant acceleration.*

Allow ECF for T from (e)(i).

[2 marks]

This question is about simple harmonic motion (SHM).

The graph shows the variation with time t of the acceleration a of an object X undergoing simple harmonic motion (SHM).



2a. Define *simple harmonic motion (SHM)*. *[2 marks]*

Markscheme

force/acceleration proportional to the displacement/distance from a (fixed/equilibrium) point/mean position;

directed towards this (equilibrium) point / in opposite direction to displacement/ distance;

Allow algebra only if symbols are fully explained.

2b. X has a mass of 0.28 kg. Calculate the maximum force acting on X. *[1 mark]*

Markscheme

0.73 (N); *(allow answer in range of 0.71 to 0.75 (N))*

2c. Determine the maximum displacement of X. Give your answer to an appropriate number of significant figures. *[4 marks]*

Markscheme

use of $a_0 = -\omega^2 x_0$;

$T = 7.9\ (\mathrm{s})$ ***or*** $\omega = 0.795$ ***or*** $\frac{\pi}{4}\ (\mathrm{rad\,s^{-1}})$; } *(allow answers in the range of T = 7.8 to 8.0 (s)* ***or*** *ω = 0.785 to 0.805 (rad s^{-1}))*

$x_0 = 4.1(1)\ (\mathrm{m})$; *(allow answers in the range of 4.0 to 4.25 (m))*

two significant figures in final answer whatever the value;

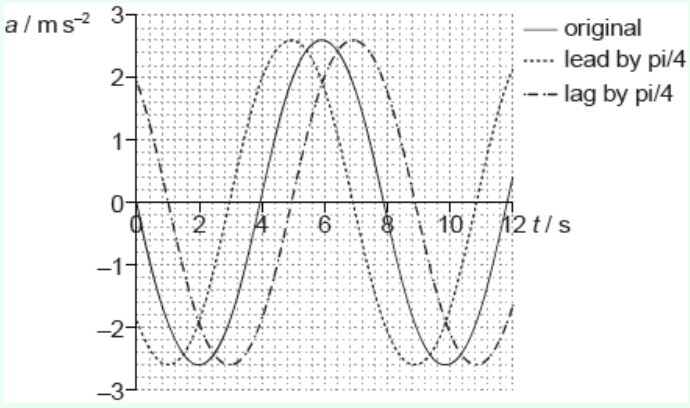
Award ***[4]*** *for a bald correct answer.*

2d. A second object Y oscillates with the same frequency as X but with a phase difference of $\frac{\pi}{4}$. Sketch, using the graph opposite, how the acceleration of object Y varies with t. *[2 marks]*

Markscheme

shape correct, constant amplitude for new curve, minimum of 10 s shown; }
(there must be some consistent lead or lag and no change in T)

lead/lag of 1 s (to within half a square by eye);

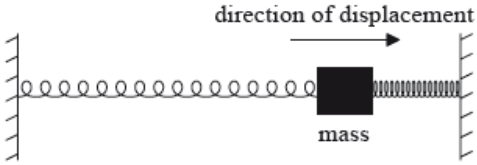


This question is in **two** parts. **Part 1** is about the oscillation of a mass. **Part 2** is about nuclear fission.

Part 1 Oscillation of a mass

A mass of 0.80 kg rests on a frictionless surface and is connected to two identical springs both of which are fixed at their other ends. A force of 0.030 N is required to extend or compress each spring by 1.0 mm. When the mass is at rest in the centre of the arrangement, the springs are not extended.

The mass is displaced to the right by 60 mm and released.



(not to scale)

3a. Determine the acceleration of the mass at the moment of release. *[3 marks]*

Markscheme

force of 1.8 N for each spring so total force is 3.6 N;

acceleration $= \frac{3.6}{0.8} = 4.5\ \text{ms}^{-2}$; *(allow ECF from first marking point)*

to left/towards equilibrium position / negative sign seen in answer;

3b. Outline why the mass subsequently performs simple harmonic motion (SHM). *[2 marks]*

Markscheme

force/acceleration is in opposite direction to displacement/towards equilibrium position;

and is proportional to displacement;

3c. Calculate the period of oscillation of the mass. *[2 marks]*

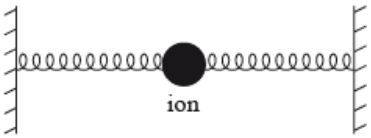
Markscheme

$\omega = \left(\sqrt{\left(\frac{a}{x}\right)} =\right) \sqrt{\frac{4.5}{60\times10^{-3}}} (= 8.66 \text{ rad s}^{-1})$;

$T = 0.73$ s;

Watch out for ECF from (a)(i) eg award ***[2]*** *for* $T = 1.0\ s$ *for* $a = 2.25 \text{ m s}^{-2}$.

The motion of an ion in a crystal lattice can be modelled using the mass–spring arrangement. The inter-atomic forces may be modelled as forces due to springs as in the arrangement shown.



The frequency of vibration of a particular ion is 7×10^{12} Hz and the mass of the ion is 5×10^{-26} kg. The amplitude of vibration of the ion is 1×10^{-11} m.

3d. Estimate the maximum kinetic energy of the ion. *[2 marks]*

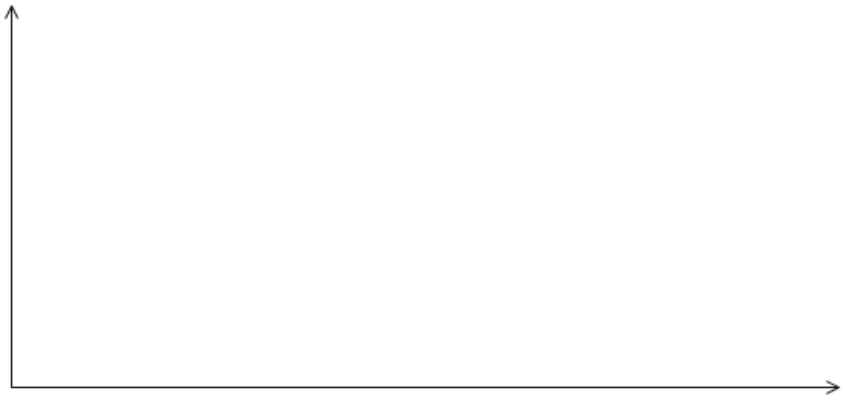
Markscheme

$\omega = 2\pi \times 7 \times 10^{12} (= 4.4 \times 10^{13} Hz)$;

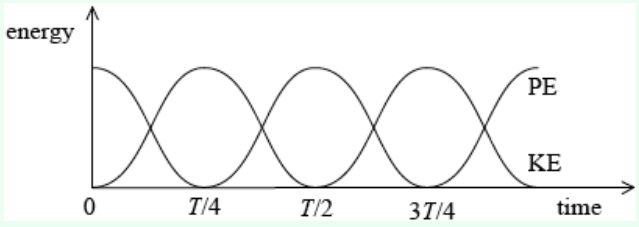
5×10^{-21} J;

Allow answers in the range of 4.8 to 4.9×10^{-21} J *if 2 sig figs or more are used.*

3e. On the axes, draw a graph to show the variation with time of the kinetic energy of mass and the elastic potential energy stored in the springs. You should add appropriate values to the axes, showing the variation over one period. *[3 marks]*



Markscheme



KE and PE curves labelled – very roughly $\cos^2$ and $\sin^2$ shapes; } *(allow reversal of curve labels)*

KE and PE curves in anti-phase and of equal amplitude;

at least one period shown;

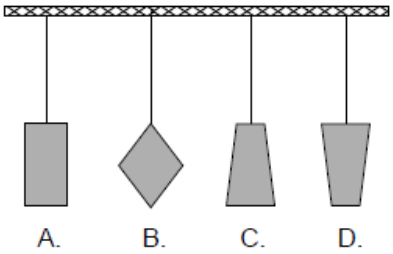
either $E_{\max}$ marked correctly on energy axis, or T marked correctly on time axis;

3f. Calculate the wavelength of an infrared wave with a frequency equal to that of the model in (b). *[1 mark]*

Markscheme

7.0×10^{12} Hz is equivalent to wavelength of 4.3×10^{-5} m;

4. The four pendulums shown have been cut from the same uniform sheet of board. They are attached to the ceiling with strings of equal length. *[1 mark]*



Which pendulum has the shortest period?

Markscheme

D

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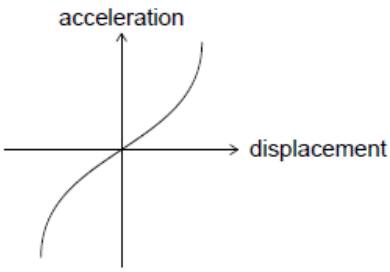
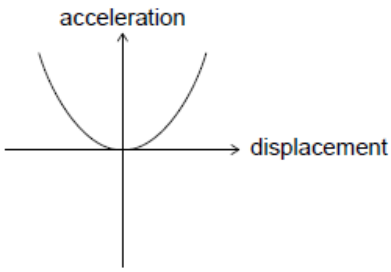
5. A mass on a spring is displaced from its equilibrium position. Which graph represents the variation of acceleration with displacement for the mass after it is released? *[1 mark]*

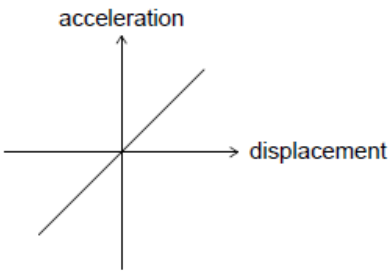
A.



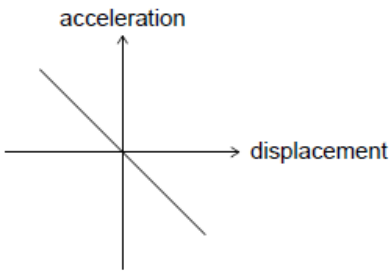
B.



C.



D.



Markscheme

D

6. An object undergoing simple harmonic motion (SHM) has a period T and total energy E. The amplitude of oscillations is halved. What are the new period and total energy of the system? *[1 mark]*

	Period	Total energy
A.	$\frac{T}{2}$	$\frac{E}{4}$
B.	$\frac{T}{2}$	$\frac{E}{2}$
C.	T	$\frac{E}{4}$
D.	T	$\frac{E}{2}$

Markscheme

C

7. A mass oscillates with simple harmonic motion (SHM) of amplitude x_0. Its total energy is 16 J. *[1 mark]*

What is the kinetic energy of the mass when its displacement is $\frac{x_0}{2}$?

A. 4 J

B. 8 J

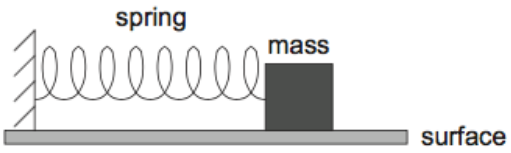
C. 12 J

D. 16 J

Markscheme

C

8. A mass is connected to a spring on a frictionless horizontal surface as shown. *[1 mark]*



The spring is extended beyond its equilibrium length and the mass executes simple harmonic motion (SHM). Which of the following is independent of the initial displacement of the spring?

A. The angular frequency of the oscillation

B. The total energy of the mass

C. The average speed of the mass

D. The maximum kinetic energy of the mass

Markscheme

A

9. An object undergoes simple harmonic motion with time period T and amplitude 0.5 m. At time $t = 0$ s the displacement of the object is a maximum. *[1 mark]*

What is the displacement of the object at time $t = \frac{3T}{4}$?

A. -0.50 m
B. 0.50 m
C. 0.25 m
D. 0 m

Markscheme

D

10. An object undergoes simple harmonic motion (SHM). The total energy of the object is proportional to *[1 mark]*

A. the amplitude of the oscillations.

B. the time period of the oscillations.

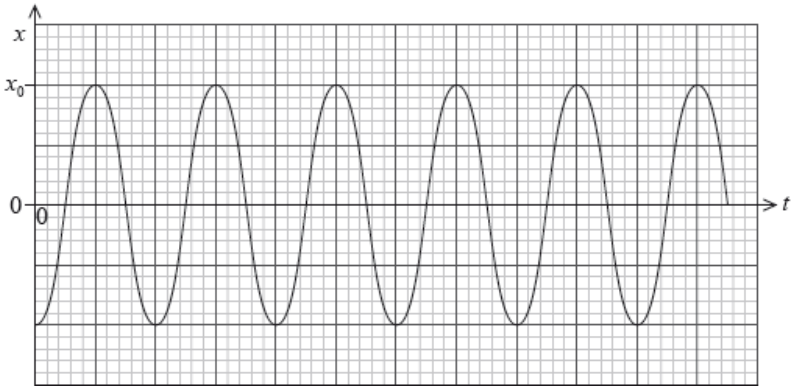
C. the frequency of the oscillations.

D. the mass of the object.

Markscheme

D

An object at the end of a spring oscillates vertically with simple harmonic motion. The graph shows the variation with time t of the displacement x. The amplitude is x_0 and the period of oscillation is T.



11. Which of the following is the correct expression for the displacement x? *[1 mark]*

A. $-x_0 \cos \frac{2\pi}{T} t$

B. $x_0 \cos \frac{2\pi}{T} t$

C. $-x_0 \sin \frac{2\pi}{T} t$

D. $x_0 \sin \frac{2\pi}{T} t$

Markscheme

A
