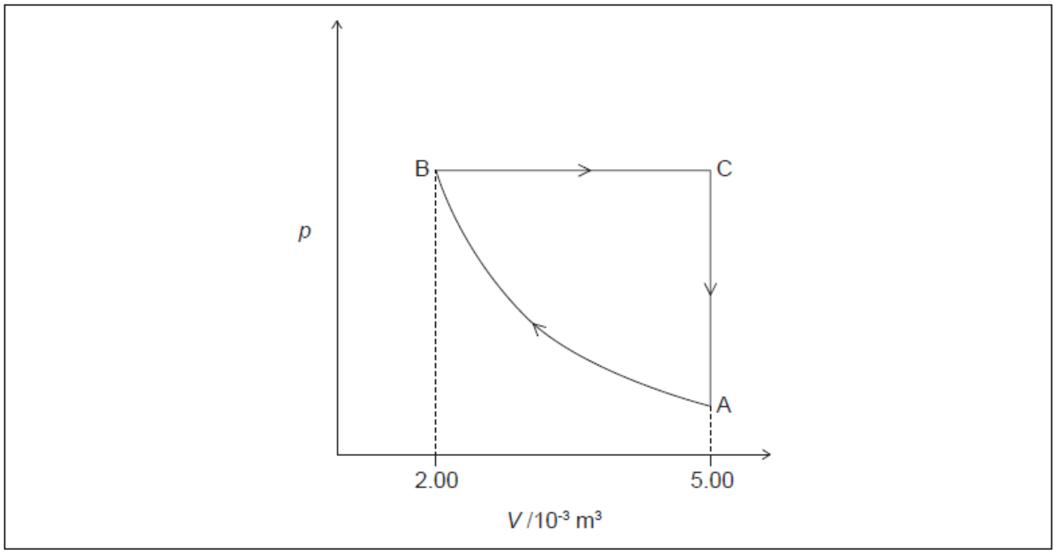
Thermodynamics *[52 marks]*

The pressure–volume (pV) diagram shows a cycle ABCA of a heat engine. The working substance of the engine is 0.221 mol of ideal monatomic gas.

diagram not to scale



At A the temperature of the gas is 295 K and the pressure of the gas is 1.10×10^5 Pa. The process from A to B is adiabatic.

1a. Show that the pressure at B is about 5×10^5 Pa. *[2 marks]*

Markscheme

«$p_1 V_1^{\frac{5}{3}} = p_2 V_2^{\frac{5}{3}}$»

$1.1 \times 10^5 \times 5^{\frac{5}{3}} = p_2 \times 2^{\frac{5}{3}}$

p_2 «$= \frac{1.1 \times 10^5 \times 5^{\frac{5}{3}}}{2.5^{\frac{5}{3}}}$» = 5.066×10^5 «Pa»

Volume may be in litres or m^3.

*Value to at least 2 sig figs, **OR** clear working with substitution required for mark.*

[2 marks]

1b. For the process BC, calculate, in J, the work done by the gas. *[1 mark]*

Markscheme

«$W = p\Delta V$»

«$= 5.07 \times 10^5 \times (5 \times 10^{-3} - 2 \times 10^{-3})$»

$= 1.52 \times 10^3$ «J»

*Award **[0]** if POT mistake.*

[1 mark]

1c. For the process BC, calculate, in J, the change in the internal energy of the gas. *[1 mark]*

Markscheme

$\Delta U = \frac{3}{2} p \Delta V = \frac{3}{2} 5.07 \times 10^5 \times 3 \times 10^{-3} = 2.28 \times 10^{-3}$ «J»

Accept alternative solution via T_c.

[1 mark]

1d. For the process BC, calculate, in J, the thermal energy transferred to the gas. *[1 mark]*

Markscheme

Q «$= (1.5 + 2.28) \times 10^3 =$» 3.80×10^3 «J»

Watch for ECF from (b)(i) and (b)(ii).

[1 mark]

The process from B to C is replaced by an isothermal process in which the initial state is the same and the final volume is $5.00 \times 10^{-3}\,m^3$.

1e. Explain, without any calculation, why the pressure after this change would belower if the process was isothermal. *[2 marks]*

Markscheme

for isothermal process, PV = constant / ideal gas laws mentioned

since $V_C > V_B$, P_C must be smaller than P_B

[2 marks]

1f. Determine, without any calculation, whether the net work done by the engine during one full cycle would increase or decrease. *[2 marks]*

Markscheme

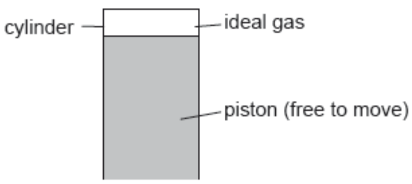
the area enclosed in the graph would be smaller

so the net work done would decrease

Award MP2 only if MP1 is awarded.

[2 marks]

A cylinder is fitted with a piston. A fixed mass of an ideal gas fills the space above the piston.



The gas expands isobarically. The following data are available.

$$\begin{aligned}
&\text{Amount of gas} &&= 243 \text{ mol}\\
&\text{Initial volume of gas} &&= 47.1 \text{ m}^3\\
&\text{Initial temperature of gas} &&= -12.0\ ^\circ\text{C}\\
&\text{Final temperature of gas} &&= +19.0\ ^\circ\text{C}\\
&\text{Initial pressure of gas} &&= 11.2 \text{ kPa}
\end{aligned}$$

2a. Show that the final volume of the gas is about 53 m ^{3}. *[2 marks]*

Markscheme

ALTERNATIVE 1

«Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$»

$V_2 = \frac{47.1 \times (273+19)}{(273-12)}$

V_2 = 52.7 «m^3»

ALTERNATIVE 2

«Using $PV = nRT$»

$V = \frac{243 \times 8.31 \times (273+19)}{11.2 \times 10^3}$

V = 52.6 «m^3»

[2 marks]

2b. Calculate, in J, the work done by the gas during this expansion. *[2 marks]*

Markscheme

W «$= P\Delta V$» $= 11.2 \times 10^3 \times (52.7 - 47.1)$

$W = 62.7 \times 10^3$ «J»

Accept 66.1×10^3 J if 53 used

Accept 61.6×10^3 J if 52.6 used

[2 marks]

2c. Determine the thermal energy which enters the gas during this expansion. *[3 marks]*

Markscheme

ΔU «$= \frac{3}{2} nR\Delta T$» $= 1.5 \times 243 \times 8.31 \times (19 - (-12)) = 9.39 \times 10^4$

Q «$= \Delta U + W$» $= 9.39 \times 10^4 + 6.27 \times 10^4$

$Q = 1.57 \times 10^5$ «J»

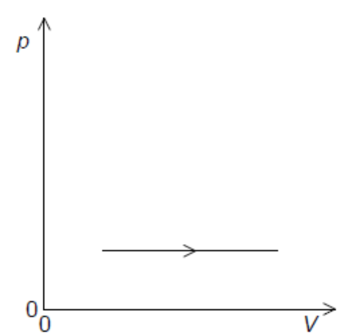
Accept 1.60×10^5 if 66.1×10^3 J used

Accept 1.55×10^5 if 61.6×10^3 J used

[3 marks]

The gas returns to its original state by an adiabatic compression followed by cooling at constant volume.

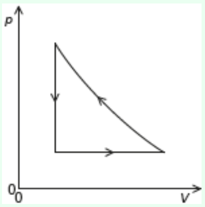
2d. Sketch, on the pV diagram, the complete cycle of changes for the gas, labelling the changes clearly. The expansion shown in (a) and (b) is drawn for you. *[2 marks]*



Markscheme

concave curve from RHS of present line to point above LHS of present line

vertical line from previous curve to the beginning



[2 marks]

A monatomic ideal gas is confined to a cylinder with volume 2.0 x 10 $^{-3}$ m^3. The initial pressure of the gas is 100 kPa. The gas undergoes a three-step cycle. First, the gas pressure increases by a factor of five under constant volume. Then, the gas expands adiabatically to its initial pressure. Finally it is compressed at constant pressure to its initial volume.

3a. Show that the volume of the gas at the end of the adiabatic expansion is approximately 5.3 x 10^{-3} m^3. *[2 marks]*

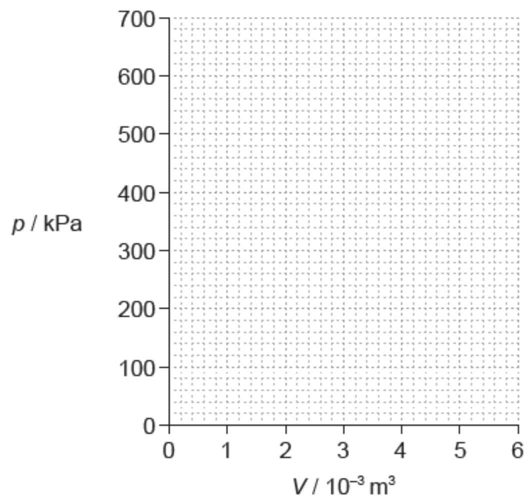
Markscheme

$$500\,000 \times \left(2 \times 10^{-3}\right)^{\frac{5}{3}} = 100\,000 \times V^{\frac{5}{3}}$$

V = 5.3 x 10^{-3} «m^3»

Look carefully for correct use of pV^{γ} = constant

3b. Using the axes, sketch the three-step cycle. *[2 marks]*



Markscheme

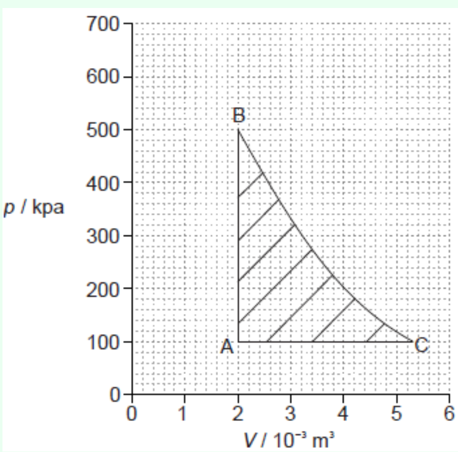
correct vertical and horizontal lines

curve between B and C

Allow tolerance ±1 square for A, B and C

Allow ECF for MP2

Points do not need to be labelled for marking points to be awarded



3c. The initial temperature of the gas is 290 K. Calculate the temperature of the gas at the start of the adiabatic expansion. *[2 marks]*

Markscheme

use of $PV = nRT$ ***OR*** use of $\frac{P}{T}$ = constant

T = «5 x 290 =» 1450 «K»

3d. Using your sketched graph in (b), identify the feature that shows that net work is done by the gas in this three-step cycle. *[2 marks]*

Markscheme

area enclosed

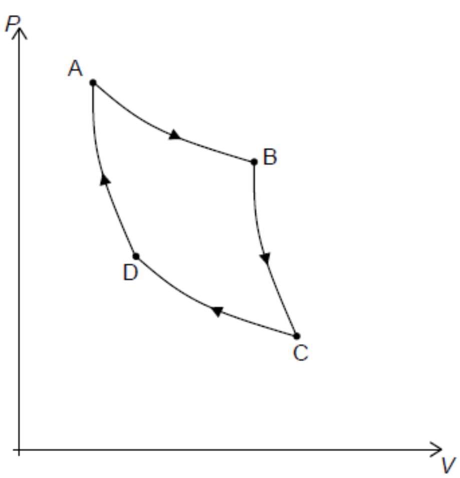
work is done by the gas during expansion

OR

work is done on the gas during compression

the area under the expansion is greater than the area under the compression

The *P*–*V* diagram of the Carnot cycle for a monatomic ideal gas is shown.



4a. State what is meant by an adiabatic process. *[1 mark]*

Markscheme

«a process in which there is» no thermal energy transferred between the system and the surroundings

[1 mark]

4b. Identify the two isothermal processes. *[1 mark]*

Markscheme

A to B ***AND*** C to D

[1 mark]

The system consists of 0.150 mol of a gas initially at A. The pressure at A is 512 k Pa and the volume is 1.20 × 10^{-3} m^3.

4c. Determine the temperature of the gas at A. *[2 marks]*

Markscheme

$T = \frac{PV}{nR}$

$T\left(= \frac{512\times10^3\times1.20\times10^{-3}}{0.150\times8.31}\right) \approx 493$ «K»

The first mark is for rearranging.

[2 marks]

4d. The volume at B is 2.30 × 10^{-3} m^3. Determine the pressure at B. *[2 marks]*

Markscheme

$P_B = \frac{P_a V_A}{V_B}$

$P_B = 267 \text{ KPa}$

The first mark is for rearranging.

[2 marks]

At C the volume is V_C and the temperature is T_C.

4e. Show that $P_B V_B^{\frac{5}{3}} = nRT_C V_C^{\frac{2}{3}}$ *[1 mark]*

Markscheme

«B to C adiabatic so» $P_B V_B^{\frac{5}{3}} = P_C V_C^{\frac{5}{3}}$ ***AND*** $P_C V_C = nRT_C$ «combining to get result»

It is essential to see these 2 relations to award the mark.

[1 mark]

4f. The volume at C is 2.90 × 10^{-3} m^3. Calculate the temperature at C. *[2 marks]*

Markscheme

$$T_C = \left(\frac{P_B V_B^{\frac{5}{3}}}{nR}\right) V_C^{\frac{-2}{3}}$$

$T_C =$ «$\left(\frac{267\times10^3\times(2.30\times10^{-3})^{\frac{5}{3}}}{0.150\times8.31}\right)\left(2.90\times10^{-3}\right)^{\frac{-2}{3}}$» = 422 «K»

[2 marks]

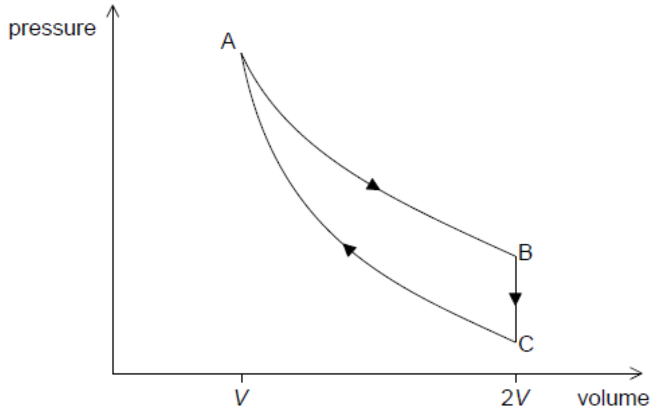
4g. State a reason why a Carnot cycle is of little use for a practical heat engine. *[1 mark]*

Markscheme

the isothermal processes would have to be conducted very slowly / OWTTE

[1 mark]

A heat engine operates on the cycle shown in the pressure–volume diagram. The cycle consists of an isothermal expansion AB, an isovolumetric change BC and an adiabatic compression CA. The volume at B is double the volume at A. The gas is an ideal monatomic gas.



At A the pressure of the gas is 4.00 x 10 ^{6} Pa, the temperature is 612 K and the volume is 1.50 x 10^{-4} m^3. The work done by the gas during the isothermal expansion is 416 J.

5a. Justify why the thermal energy supplied during the expansion AB is 416 J. *[1 mark]*

Markscheme

$\Delta U = 0$ so $Q = \Delta U + W = 0 + 416 = 416$ «J»

Answer given, mark is for the proof.

[1 mark]

5b. Show that the temperature of the gas at C is 386 K. *[2 marks]*

Markscheme

ALTERNATIVE 1

use $pV^{\frac{5}{3}} = c$ to get $TV^{\frac{2}{3}} = c$

hence $T_{\rm C} = T_{\rm A}\left(\frac{V_{\rm A}}{V_{\rm C}}\right)^{\frac{2}{3}} = 612 \times 0.5^{\frac{2}{3}} = 385.54$

«T_C ≈ 386K»

ALTERNATIVE 2

$P_{\rm C}V_{\rm C}{}^{\gamma} = P_{\rm A}V_{\rm A}{}^{\gamma}$ giving P_C = 1.26 x 10^6 «Pa»

$\frac{P_{\rm C}V_{\rm C}}{T_{\rm C}} = \frac{P_{\rm A}V_{\rm A}}{T_{\rm A}}$ giving $T_{\rm C} = 1.26 \times \frac{612}{2} = 385.54$ «K»

«T_C ≈ 386K»

Answer of 386K is given. Look carefully for correct working if answers are to 3 SF.

There are other methods:

Allow use of P_B = 2 x 10^6 «Pa» and
$\frac{P}{T}$ is constant for BC.

Allow use of n = 0.118 and T_C = $\frac{P_{\rm C}V_{\rm C}}{nR}$

[2 marks]

5c. Show that the thermal energy removed from the gas for the change BC is approximately 330 J. *[2 marks]*

Markscheme

$Q = \Delta U + W = \frac{3}{2}\frac{P_{\rm A}V_{\rm A}}{T_{\rm A}}\Delta T + 0$

$Q = \frac{3}{2} \times \frac{4.00 \times 10^6 \times 1.50 \times 10^{-4}}{612} \times (386 - 612)$

«–332 J»

Answer of 330 J given in the question.
Look for correct working or more than 2 SF.

[2 marks]

5d. Determine the efficiency of the heat engine. *[2 marks]*

Markscheme

$e = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{412-332}{416}$

e = 0.20

Allow $\frac{416-330}{416}$.

Allow e = 0.21.

[2 marks]

5e. State and explain at which point in the cycle ABCA the entropy of the gas is the largest. *[3 marks]*

Markscheme

entropy is largest at B

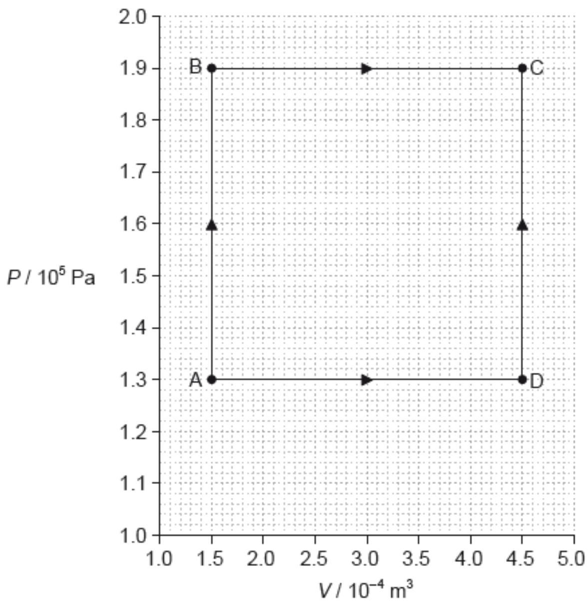
entropy increases from A to B because T = constant but volume increases so more disorder ***or*** $\Delta S = \frac{Q}{T}$ and $Q > 0$ so $\Delta S > 0$

entropy is constant along CA because it is adiabatic, $Q = 0$ and so $\Delta S = 0$
OR
entropy decreases along BC since energy has been removed, $\Delta Q < 0$ *so* $\Delta S < 0$

[3 marks]

This question is about an ideal gas.

The graph shows how the pressure P of a sample of fixed mass of an ideal gas varies with volume V.



The temperature of the gas at point A is 85 °C. The gas can change its state to that of point C either along route ABC or route ADC.

6a. Calculate the temperature of the gas at point C. *[3 marks]*

Markscheme

use of $\frac{PV}{T} = \text{constant}$ ***or*** use of $T \propto PV$ ***or*** via intermediate calculation of n in $PV = nRT$;

$\frac{1.95}{358} = \frac{8.55}{T_c}$ ***or*** $n = 6.55 \times 10^{-3}\ (\text{mol});\}$ *(allow power of ten omission provided same omission on both sides)*

1570 K ***or*** 1300 °C;

*Award **[3]** for a bald correct answer ignoring small rounding errors.*

*Omitting conversion to Kelvin yields answer of 373 – award **[2 max]** as one error.*

*Award **[1 max]** if 273 subsequently added (to give 646).*

6b. Compare, without any calculation, the work done and the thermal energy supplied along route ABC **and** route ADC. *[3 marks]*

Markscheme

same temperature change so same change in internal energy/ ΔU;

work done along ABC is larger/ADB is smaller because area under ABC is greater than area under ADC/ΔV same in both, P greater for ABC so $P\Delta V$ also greater for ABC;

because $\Delta Q = \Delta U + W$ thermal energy transferred is greater for route ABC/smaller for route ADB;

Must see reference to first law for MP3.