FORCED VIBRATIONS AND RESONANCE

Engineering: B.4, pages 582-588

Free Vibration

- A mass is free to oscillate
- No external forces acting on the mass
- Amplitude, frequency, period…all constant

- **Natural Frequency** (f_0)
  - The frequency of the free vibrations of a system

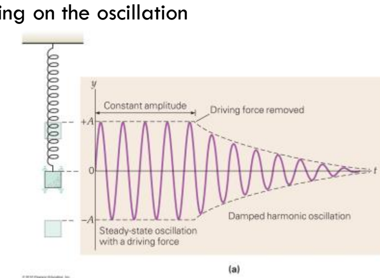
What if there IS an external force?

- Resistive forces tend to increase in magnitude with increased speed of the system
  - Always act in opposite direction of motion
- Maximum resistive force
  - → at equilibrium point (highest speed)
- Zero resistive force
  - → at amplitude (speed = 0)

- To oppose the resistive force, the oscillating system does work, transfers energy out of the system
- Amplitude decreases

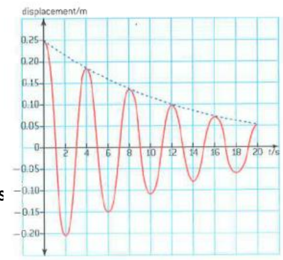
Damping

- The effect on a system when a resistive force is acting on the oscillation



Under-Damping

- Light damping
- Rate of damping (the rate of decrease of the amplitude) is an exponential decay
- The ratio of the amplitudes at half-period intervals remains constant
- Frequency remains constant



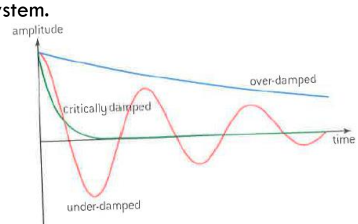
▲ Figure 1 Under-damped vibration.

Critical Damping

- "Heavy damping"
- System returns to equilibrium position in shortest possible time
- System does NOT oscillate past the equilibrium point before stopping
- Examples: car suspensions; some fire doors; shock absorbers of all sorts

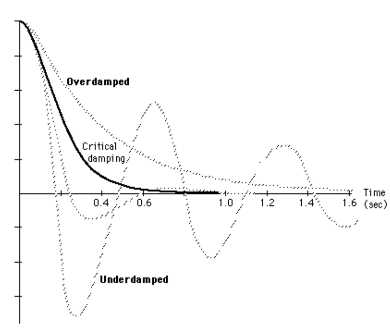
Over-Damped

- System stops oscillating quickly, but takes longer to get to the equilibrium point than a critically damped system.



▲ Figure 2 Variation of amplitude with time for different degrees of damping.

Graphical comparison of damping effects

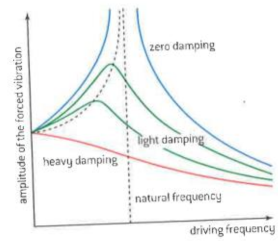


Forced Vibrations

- Those vibrations that occur when an external force is applied at a regular frequency so that the system vibrates at the same frequency as the force
- If the frequency of the applied force is the same as (or close to) the natural frequency of the oscillating system, the amplitude will increase
- If the frequency of the applied force is significantly different than the oscillation's frequency, the amplitude will possibly decrease
- External force = **Driving Force**

Resonance

- Occurs when **driving force** is applied at a frequency that matches the natural frequency of the oscillating system.
- When damping occurs, effects are minimized.
- When no damping occurs, maximum amp.



▲ Figure 4 Effect of damping on resonance.

Examples

- **Good (beneficial) Resonance**
  - Musical instruments
  - Earthquake counter-balance (damping to minimize effects!)
  - Quartz crystals in precision timing devices
  - LASERs
  - Cooking in microwave ovens
- **Destructive Resonance**
  - Building destruction during earthquakes
  - Tacoma Narrows Bridge collapse
  - Airplane wings (esp. small aircraft) when strong winds pass over them: https://www.youtube.com/watch?v=iTFZNrTYp3k
  - https://www.youtube.com/watch?v=ImSuZjvkATw
  - Ground Resonance:
    - https://www.youtube.com/watch?v=-LFLV47VAbI
    - https://www.youtube.com/watch?v=vTRuWgoEFxo

Q Factor

- "Quality" Factor
- The quantifiable criterion that allows us to asses the amount of resonance that will occur

$$Q = 2\pi \frac{Energy\ stored}{Energy\ dissipated\ per\ cycle}$$

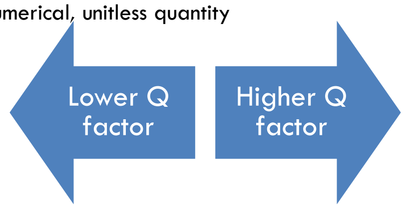
- OR

$$Q = 2\pi(resonant\ frequency)\left(\frac{energy\ stored}{power\ loss}\right)$$

See page 585 for an explanation of how these are equivalent expressions. ☺

Q factor

- Numerical, unitless quantity



High damping　　Light Damping

Q Factor for Mechanical Oscillators

A sample list of Q factor values. These are not exact quantities for every situation.

Oscillator	Q Factor
Critically damped door	0.5
Mass on spring	50
Simple Pendulum	200
Oscillating quartz crystal	30000

Sample problem

- An electrical pendulum clock has a period of 1.0 s. An electrical power supply of 25 mW maintains its constant amplitude. As the pendulum passes its equilibrium position it has kinetic energy of 40.0 mJ.
  - **Turn and talk: how do these quantities apply to the Q factor relationship?**
  - **Calculate** the Q factor for this pendulum clock

Reading assignment:

- Read about Barton's Pendulums and the Nature of Science examples of resonance on pages 586-587 of your textbook.